

Proof Without Words: Inequalities for Two Numbers Whose Sum Is One

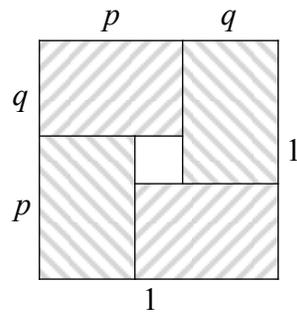
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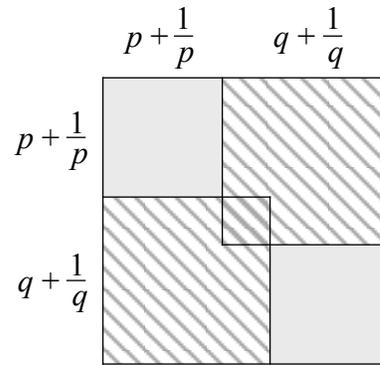
$$p, q > 0, \quad p + q = 1 \Rightarrow \frac{1}{p} + \frac{1}{q} \geq 4 \text{ and } \left(p + \frac{1}{p}\right)^2 + \left(q + \frac{1}{q}\right)^2 \geq \frac{25}{2}.$$

Proof.

(a)



(b)



$$(a) \quad 1 \geq 4pq \Rightarrow \frac{1}{p} + \frac{1}{q} \geq 4,$$

$$(b) \quad 2\left(p + \frac{1}{p}\right)^2 + 2\left(q + \frac{1}{q}\right)^2 \geq \left(p + \frac{1}{p} + q + \frac{1}{q}\right)^2 \geq (1 + 4)^2 = 25. \quad \blacksquare$$

REFERENCE

1. Problema 20/3(c), XX Olimpiada Matemática Española (1983-84), *Olimpiada Matemática Española en el Siglo XX*, Real Sociedad Matemática Española, Madrid, 2000.